Low-CNR inverse synthetic aperture LADAR imaging demonstration with atmospheric turbulence

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Russell Trahan, Bijan Nemati, Hanying Zhou, Michael Shao, Inseob Hahn, William B. Schulze

Presented by Russell Trahan
Summary

Goals:

• Demonstrate ISAL functionality in photon-starved conditions.
• Find a metric that can predict the success/failure of PGA based on the return signal strength.

Outline:

• Testbed hardware setup and data processing
  • Basic setup for low-CNR
  • Atmospheric turbulence synthesis
  • Data pipeline

• CNR
  • CNR definition for a single range-bin (including detector noise)
  • Various metrics based on CNR
  • Image quality metric to compare to metrics based on CNR

• Experimental Data
  • High CNR functionality tests
  • Low CNR imaging examples showing PGA failure at mean CNR=\sim0.25
Testbed Hardware Setup and Data Processing
Transceiver / Target Layout

Target

PZT Target Rotation Stage

Cross Range

Range

Top View

Line Target

Side View

Circle Target

CNR Derivation

Experimental Data

Conclusion
Transceiver Assembly

To Target

Local Oscillator

Receiver

Transmitter
Transmitter Designs

- No atmospheric turbulence
  - Fiber termination and collimating lens

- Atmospheric turbulence
  1. Fiber Termination
  2. Collimating Lens – collimate light from fiber
  3. Iris – truncate Gaussian beam to FWHM
  4. Focusing Lens – focus collimated light through the phase wheel
  5. Phase Wheel – introduce phase error
  6. Speckle Image – focal point of focusing lens
  7. Magnification Lens – magnify the speckle image onto the target
Testbed Overview

Phase Wheel
Transmitter
Receiver
LO
Focus Lens
Mag. Lens

CNR Derivation
Experimental Data
Conclusion
Our best results came from starting the window at 75% of the cross range extent, allowing $\bar{\phi}$ to converge to nearly zero, then decreasing window size by 25%. Repeat until window is ~10 pixels in cross range.

Over-sampling in range or including range-bins with very low CNR shouldn’t influence the phase increments. Simply includes noise in summation.
CNR Derivation and Image Quality Metrics
CNR Definition

- CNR is defined as
  \[ \text{Estimate of carrier strength} \text{ StdDev of estimate of carrier strength} \]

- Measurement can be modeled as
  \[ \eta_d \sqrt{\eta_s N_L \tilde{N}_S} \exp(i\varphi) = \eta_d \sqrt{\eta_s N_L N_S} \exp(i\varphi_s) + N(0, \sigma_{SN}^2) + N(0, \sigma_{NEP}^2) \]
  - The carrier for a single range bin is \( \eta_d \sqrt{\eta_s N_L N_S} \exp(i\varphi_s) \)
  - Shot noise variance is \( \sigma_{SN}^2 \approx \eta_d N_L / 2 \)
  - Detector NEP noise variance is \( \sigma_{NEP}^2 = \frac{P_{NEP}^2 \tau}{2h^2 \nu^2} \)

- Model is used to estimate the carrier strength and its variance
  \[ \text{CNR} = \frac{\langle N_L \tilde{N}_S \rangle}{\sqrt{\text{var}(N_L \tilde{N}_S)}} = \frac{N_S}{\sqrt{\frac{2N_s}{\eta_d \eta_h} + \frac{1}{\eta_d^2 \eta_h^2 N_L} + \frac{4\sigma_{NEP}^2}{\eta_d^2 \eta_h^2 N_L} + \frac{4\sigma_{NEP}^4}{\eta_d^4 \eta_h^4 N_L}}} \]
  \[ \approx \frac{N_S}{\sqrt{\frac{2N_s}{\eta_d \eta_h} + \frac{1}{\eta_d^2 \eta_h^2}}} \text{ for } N_s \gg 1/\eta_d \eta_h \]
  \[ \approx \eta_d \eta_h N_s \text{ for } N_s \ll 1/\eta_d \eta_h \]

Quality metrics based on pre-PGA data:

- # Photons in each range-bin
  Maximum, Mean, Sum, Sum of squares
- CNR of mean photons per range-bin
- CNR of each range-bin
  Maximum, Mean, Sum, Sum of squares
- Phase progression
  Variance of each range-bin
  Maximum, Mean, Sum, Sum of squares

Quality metric based on post-PGA result:

- Image Contrast-to-Noise Ratio
  \[ C = \frac{\text{mean(foreground)} - \text{mean(background)}}{\text{stddev(background)}} \]
- Foreground region is determined based on a priori knowledge of the target.
- PGA performance cannot be assessed as \( C \) decreases past 1.

Primary Question:
What quality metric has a consistent value at the threshold where PGA doesn’t work?

Immediate Question:
What quality metric has a consistent value when the image contrast-to-noise ratio is 1?
Contrast depends on Cross-Range Extent

Considering only a single range bin and a consistent CNR:

- The image contrast is inversely proportional to the number of cross-range bins populated by the target.

- Parseval’s Theorem: \[ \sum_{n=0}^{N-1} |P_n|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |p_k|^2 \]

- Sum of a single range-bin’s magnitude over all pulses must equal the mean of the cross-range pixel values.
  - If a single cross-range pixel is filled by the target, contrast will be high.
  - If several cross-range pixels are filled by the target, contrast will be low.

*This idea is confirmed in the experimental data presented later.*
Imaging Examples
~2m Range to Target

Cross Range

Range

Top View

Line Target

Circle Target

Side View

Range

Jet Propulsion Laboratory
California Institute of Technology
Sample Low CNR Result

Testbed

CNR Derivation

Experimental Data

Conclusion

Contrast: 1.9

# LO Photons per pulse: 5.05e+12

# Range Bins: 33.9

# Photons per Range Bin:
  - Max: 1.92
  - Mean: 0.55
  - Sum: 18.54
  - Sum of sqr: 18.52

CNR of Mean Photons per Range Bin: 0.27

CNR of Active Range Bins:
  - Max: 0.66
  - Mean: 0.24
  - Sum: 8.15
  - Sum of sqr: 3.14

Difference
Front View

<table>
<thead>
<tr>
<th>Tested Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chirp Rate</td>
<td>2THz/s</td>
</tr>
<tr>
<td>Pulse Length</td>
<td>34 ms</td>
</tr>
<tr>
<td>Acq Time</td>
<td>60 s</td>
</tr>
<tr>
<td>Mean CNR</td>
<td>2.76</td>
</tr>
</tbody>
</table>
Contrast vs Mean CNRs

**Testbed** ●●●●●● CNR Derivation ●●● Experimental Data ●●●●○ Conclusion ○

- **Line**
- **Area**
- **Top View**

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Contrast vs Mean CNRs

<table>
<thead>
<tr>
<th>Line</th>
<th>Area</th>
<th>Line w/turbulence</th>
</tr>
</thead>
</table>

Graph showing the relationship between Contrast and Mean CNR with different lines and areas.
Low Mean CNR Images

- Line Target (top row)
- Area Target (bottom row)

Contrast:
- No Turbulence: 5.9, Mean CNR: 1.32
- No Turbulence: 3.2, Mean CNR: 1.07
- Turbulence: 1.3, Mean CNR: 0.31
- Turbulence: 0.84, Mean CNR: 0.31

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Conclusions

• Testbed build to perform ISAL studies
  • Short 2m or long 400m range-to-target
  • Synthesized atmospheric turbulence
  • High and very low CNR capabilities

• CNR Derivation
  • Rigorous derivation of CNR for a single range-bin
  • Quality metric for overall signal: “Mean CNR”
  • Quality metric for image: Contrast-to-Noise Ratio

• Experimental Results
  • Target cross-range extent decreases image contrast (for constant CNR)
  • PGA can work for simple images down to ~0.25 CNR
  • Atmospheric turbulence raises minimum CNR threshold to ~0.75
References

Backup Slides
Photon Count Estimation

- Detector DC voltage determines local oscillator photon count:

\[ P_L = \frac{V_{DC}}{G_{DC}} \implies N_L = \frac{P_L \tau}{E_{ph}} \]

- The mean one sided PSD: \( (j^{th} \text{ voltage measurement in the } k^{th} \text{ pulse}) \)

\[ P_u = \frac{2}{N_v N_p \delta f} \sum_{k=0}^{N_v-1} \sum_{j=0}^{N_p-1} V_{j,k} \exp\left(-i2\pi \frac{ju}{N_v}\right) \], \( u = [0, N_v - 1] \)

- The number of photons in each range bin is given by:

\[ P_{Het} = \sqrt{2\delta f \left( P_u - P_{BG} \right)} \implies P_{Het} = \frac{P_{Het}^2}{4P_L} \implies N_S = \frac{P_{Het} \tau}{E_{ph}} \]
CNR Derivation

- Total power at detector due to an E field is related to the mean field amplitude:
  \[ P_d = \int \frac{1}{2} |E\exp(2\pi it + i\varphi)|^2 dA = \frac{\hbar c N}{\lambda \tau} = \frac{1}{2} A_d \overline{E}^2 \]
  \[ \overline{E}^2 = \frac{2\hbar c N}{\lambda A_d \tau} \]

- Detector output current due to single range element:
  \[ I_d = \eta_d e \int \frac{1}{2} \left| E_i \exp\left(2\pi i\left(f_0 + \frac{1}{2} \hat{f} t \right) + E_s \exp\left(2\pi i\left(f_0 + \frac{1}{2} \hat{f} (t + \Delta t) \right) \right) + i\varphi \right| dA \]
  \[ = \eta_d e \left[ \frac{1}{2} A_d \overline{E}_i^2 + \frac{1}{2} A_d \overline{E}_s^2 + A_s \sqrt{\eta_s} \overline{E}_i \overline{E}_s \cos\left(2\pi \Delta t + \varphi_s \right) \right] \]
  \[ = \eta_d e \frac{N_s + N_s}{\tau} + 2\eta_s \frac{\eta_s N_s}{\tau} \cos\left(2\pi \Delta t + \varphi_s \right) \]

- DFT of 2M samples of \( I_d \) at the carrier frequency:
  \[ D(\Delta f) = \frac{\tau}{2M} \sum_{m=0}^{2M-1} 2\eta_d e \frac{\sqrt{\eta_s N_s}}{\tau} \cos\left(2\pi \Delta f_m + \varphi_s \right) \exp\left(-2\pi i \Delta f_m \right) \]
  \[ = \eta_d e \frac{\sqrt{\eta_s N_s}}{\tau} \exp\left(i\varphi_s \right) \]

- Measured quantity is expected number of signal photons plus complex noise:
  \[ \eta_d \sqrt{\eta_s N_s} \exp(i\varphi) = \eta_d \sqrt{\eta_s N_s} \exp(i\varphi) + N(0, \sigma_{SN}^2) + N(0, \sigma_{NEP}^2) \]

- Measurement has a variance due to shot noise:
  \[ \sigma_{SN}^2 = \eta_d \frac{N_s + N_s}{2} \approx \eta_d \frac{N_s}{2}, \quad N_s \gg N_s \]

- Measurement has variance due to detector noise
  \[ \sigma_{NEP}^2 = \frac{1}{2} \left( \frac{P_{NEP} \sqrt{\tau}}{\hbar c} \right)^2 = \frac{P_{NEP}^2 \lambda^2 \tau}{2\hbar^2 c^2} \]

- CNR is defined as
  \[ \frac{\text{Estimate of carrier strength}}{\text{StdDev of estimate of carrier strength}} \]

\( A_d \): Detector area
\( \eta_d \): Detector efficiency
\( e \): Electron charge
\( G \): Detector Gain
\( \eta_s \): Heterodyne efficiency
\( N_s \): # LO photons per pulse
\( n \): # measured photons
\( N_s \): # range bins
\( N_s \): # signal photons per pulse
\( h \): Plank's constant
\( \tau \): Pulse time

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CNR Derivation (cont.)

- **CNR is defined as**
  
  \[
  \text{CNR} = \frac{\text{Estimate of carrier strength}}{\text{StdDev of estimate of carrier strength}}
  \]

- **Measurement gives number of detected photons** \( \tilde{N}_S \).

  \[
  \eta_d \sqrt{\eta_s \tilde{N}_s} \exp(i\phi) = \eta_d \sqrt{\eta_s \tilde{N}_s} \exp(i\phi) + N(0, \sigma_{SN}) + N(0, \sigma_{NEP})
  \]

  \[\sigma_{SN}^2 \approx \eta_d \frac{N_L}{2}\]

  \[\sigma_{NEP}^2 = \frac{P_{NEP}^2}{2h\nu^2}\]

- **Second moment gives estimate of** \( \tilde{N}_S \)

  \[
  \text{var}\left(\eta_d \sqrt{\eta_s \tilde{N}_s} \exp(i\phi)\right) = \text{var}\left(\eta_d \sqrt{\eta_s \tilde{N}_s} \exp(i\phi)\right) + \text{var}(N(0, \sigma_{SN})) + \text{var}(N(0, \sigma_{NEP}))
  \]

  \[
  \langle N_L \tilde{N}_s \rangle = \langle N_L N_S \rangle + 2 \frac{\sigma_{SN}^2 + \sigma_{NEP}^2}{\eta_d \eta_h}
  \]

- **Fourth moment gives variance of** \( \tilde{N}_S \)

  \[
  \text{var}(\tilde{N}_s) = \langle \tilde{N}_s^2 \rangle - \langle \tilde{N}_s \rangle^2 = \frac{4N_S}{\eta_d \eta_s N_L} \left(\sigma_{SN}^2 + \sigma_{NEP}^2\right) + \frac{4}{\eta_d \eta_s N_L} \left(\sigma_{SN}^2 + \sigma_{NEP}^2\right) + \frac{8 \sigma_{SN}^2 \sigma_{NEP}^2}{\eta_d \eta_s N_L}
  \]

  \[
  = \frac{2N_S}{\eta_d \eta_h} \left(\frac{1}{\eta_d \eta_s N_L} + \frac{4N_S \sigma_{NEP}^2}{\eta_d \eta_s N_L} + \frac{4\sigma_{SN}^2}{\eta_d \eta_s N_L} + \frac{4\sigma_{NEP}^2}{\eta_d \eta_s N_L}\right)
  \]

- **CNR**

  \[
  \text{CNR} = \frac{\langle N_L \tilde{N}_s \rangle}{\sqrt{\text{var}(N_L \tilde{N}_s)}} = \frac{N_S}{\sqrt{\eta_d \eta_s + \frac{1}{\eta_d \eta_s} + \frac{4N_S \sigma_{NEP}^2}{\eta_d \eta_s N_L} + \frac{4\sigma_{NEP}^2}{\eta_d \eta_s N_L} + \frac{4\sigma_{NEP}^2}{\eta_d \eta_s N_L}}
  \]

\[\text{where Eq}s. (5) \text{ and } (6) \text{ have been used. Taking } \eta_s = \eta_d = 1 \text{ in the first approximation shows that the best possible CNR of heterodyne detection is a factor of } \sqrt{2} \text{ below the best possible CNR of direct detection.}

\[\text{For } N_S << 1(\eta_d\eta_s), \text{ CNR}_{\text{DL}} \text{ is proportional to the number of photons detected, rather than to the square of the detector efficiency.} \]

Contrast vs Mean & Max CNR

- Line
- Area

Graphs showing contrast vs mean and max CNR.