High Precision Focal Plane Astrometry
... and possibilities for the HDST

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Sept 10, 2015
Overview

• The calibration techniques we are developing for WFIRST can be also used on HDST, turning it into a revolutionary astrometric instrument

• Estimated astrometric accuracy is 10’s of nano arcseconds

• Two examples of what such a capability allows us to do
  – Parallaxes of galactic neighborhood out to 100 Mpc
  – Astrometric characterization of sun like stars out to 50 pc
    – Masses, orbit parameters, inclinations

  • Broad search for true exo-Earths around sun like stars
    – Thousands of stars can be searched with a one year allocation

  • Precision targeted investigations of RV and Transit objects of interest
    – Modest demand: e.g. for a target 30 pc away, 100 observations totaling < 6 hrs
Focal Plane Astrometry

• Focal Plane Astrometry is in principle simple:
  – Distances between stellar images is directly related to angular distances on the sky

• The most important sources of error are:
  1. Photon noise
  2. Astrophysical noise (sun spots)
  3. Focal Plane imperfections
  4. Optical System field distortions
  5. Image position estimation

🌟: What we are addressing here
The Dominant Systematic Errors

- There are three categories of errors that have to be calibrated to enable μas astrometry telescopes

1. **Focal plane imperfections** *(detector errors)*
   - Pixels are not on a perfect regular grid
   - Their QE’s are different and not necessarily flat within a pixel

2. **Non-common footprints and field distortions** *(optical errors)*
   - Light from different stars in the field falls on different parts of the focal plane
   - Wavefront errors coupled with non-common footprints give rise to field distortions

3. **Image position estimation errors** *(algorithms)*
   - Centroiding algorithms that assume a PSF can have systematic errors when the assumed PSF is not the same as the true PSF.

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On a TMA telescope, largest beamwalk occurs on the tertiary mirror.
The Yard Stick: Fringes from Two Fibers

- The wavefront from a single mode fiber is the closest thing to a perfect sphere made by Humans.
  - Any error is in the flatness of the glass
  - A 2 inch optic can be polished to $\lambda/20$ p-v surface
  - Assuming a surface error power spectrum that goes as $f^{-2.5}$, on a 4 um scale the surface can potentially be flat to $\sim 10^{-5} \lambda$
  - The intensity distribution is a Gaussian

- Interference between two spherical wavefronts produces hyperbolic fringes (approximately straight)

Laser light in 1 fiber is freq shifted with respect to the other to provide “moving fringes” ~ a few Hz the detector is read out with ~10 reads/\( \lambda \) motion of the fringes

AOM’s
As frequency shifters

AOM driven by 2 RF Sources 40.000,000 Mhz 40.000,003 Mhz

\[ \Delta f = 3,000,000 \pm 1 \text{uHz} \]
Detector Calibration Testbed

- Calibrate the focal plane with laser metrology
  - Pixel QE, pixel location, intra-pixel QE(x,y). On orbit only update pixel location (x,y).
- Rather than fit the CCD data to a “reference” PSF, derive the true on orbit PSF with Nyquist sampled images and the math to incorporate imperfect pixels into the fit.

Conceptually, the idea is that with laser calibration of the detector, we’re measuring the position of a star with respect to the laser fringes, not pixel locations.

All components of the metrology system are common components used in fiber optic communications. Many, perhaps all are flight qualified. All components are either totally passive (e.g. fiber) or have very high mechanical resonances (shock insensitive) and are radiation insensitive.
Measuring Intra-pixel QE variations

- The fringe pattern from interfering light from two optical fibers is a sine wave.
- The recorded image is the product of the sine wave with the QE of the detector.
- We’re measuring the OTF of every pixel in detector.

Current setup has 7 fibers, (used 2 at a time) 21 different baselines in different orientations and fringe spacings.

Laser fringe calibration advantages
All pixels are calibrated at the same time
Fringe geometry accurate across the whole focal plane (pixel spacing accurate 1e-9)
Calibrating Field Distortion

- Diffractive grating generates a field of stars (Guyon Technique)
  - In lab via grating; on NRO via dots on primary
- Measure field distortion from dot location gradients

![Diagram of calibration setup]

Data (preliminary) magnified by 50x
Mosaic Motion and Field Distortion

- GAIA will provide positions for ~50 million stars with ~10 uas accuracy.
  - (and ~1e9 stars to ~100uas)
- Several different types of focal plane change

Chip distortion 40mm chip *1e-5 CTE*0.01K
~ 4nm expansion of 1 chip ~40 uas
Thermal gradients within a chip will produce more complex geometric distortion of the pixel location. Metrology would measure position of every pixel to 0.1nm.

If chip distortion and mosaic distortion are arbitrarily complex, it can not be separated from optical distortion. But if chip/mosaic distortion can be modeled with N parameters << #GAIA stars in the field, it may be separable. But metrology of the focal plane greatly simplifies distortion calibration.
We had MDL make a mask, representative of a globular cluster 47 Tuc. 

~20,000 stars over ½ deg.

Move the star field ~20% of the FOV to ~5 positions.

Model field distortion as a polynomial (or other basis function) and solve for the model coef.

Fortunately, WFIRST AFTA telescope optics will be very stable (to meet coronagraph requirements) distortion calibration repeated every few days/weeks.
PSF Modeling and Nyquist Sampling

- Simple centroid estimators \((A - B)/(A + B)\) (quad cell centroid) and center of gravity centroid estimators \(\sum i \cdot I(i)/\sum I(i)\) are not used for precision astrometry. (such as on HST)

- The normal approach is to perform a least squares fit of a pixelated model to the data. The model PSF (often an analytical model) is shifted in X, Y and intensity until the data and the model have the smallest different (Least sq)
  - Problems:
    - The PSF depends on the wave front errors caused by imperfect optics.
    - The PSF changes across the FOV.
    - The PSF depends on the color of the star.
Sampling Theorem Guarantees complete representation of Nyquist-sampled PSF

Asymmetrical PSF (exaggerated)

Because these images are Nyquist sampled any one of these can be used to reconstruct the true optical PSF.

For astrometry, shift the “true” psf until there is a least sq-fit of the pixelated true PSF with the data.
Demonstrating Astrometry at 4e-5 pixel

- Combination of focal plane calibration and improved PSF position estimation is used for a complete astrometry demonstration
  - 3 airy spots were moved them across 3 pixels of a calibrated CCD
  - Total of 30 positions, sub-averaged into 3 groups of 10
  - The separation between A, B was constant to 1.2e-4 pixels at each of the 10 positions.
  - After averaging 10 positions:
    - The separation agreed to ~10^{-5} \lambda/D = 4e-5 pixels.

Achieved average error of 9e-6 \lambda/D

PSF oversampled 4pix/(\lambda/D)
Calibration Benefits to WFIRST

- Astrometry at ~1~10uas
- photometry at 10-4~10-5. (can’t do astrometry at 1e-5 pix without photometry)
  - This is Kepler level photometry (if photon statistics allow)
- Correction of asymmetrical PSFs in images of Galaxies for Weak Lensing
- If the spatially varying PSF also changes in time (over weeks/months) that can be tracked as well.
Dynamical Parallaxes of Galaxies

- HDST can obtain distances to nearby galaxies using *Dynamic Parallax*:
  - Choose bright, $O$ stars in another galaxy
  - Measure proper motion through astrometry
  - Measure tangential velocity
    - Radial velocity
    - Oblateness of galaxy
  - Average over > 2000 $O$ stars within the distant galaxy
    - all target stars simultaneously in each epoch
- HDST’s large aperture enables high precision astrometric measurements of extreme faint objects

\[
O \left( \frac{200 \text{ km}}{s} \right) \\
\delta \theta_{\text{ast}} \frac{t_2 - t_1}{t_2 - t_1} \\
O(50 \text{ Mpc})
\]

\[
d = \frac{\nu_{tan}}{\delta \theta_{\text{ast}}}
\]
Estimated SNR for 50 Mpc distant galaxy

- Expect thousands of $O$-stars in one galaxy; assume 2000 tracked

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On a galaxy 50 Mpc out, photon noise limit is $\sim$ 5% distance measurement with 2 hrs of integration over 2 epochs 4 years apart
Exo-Earth Detection with HDST

• The large collecting aperture of the HDST makes astrometry at tens of nano arcseconds possible

• True Exo-Earths are terrestrial Earth mass planets in the habitable zones of stable, luminous, sun-like stars
  – G stars like the sun are preferred targets for true Exo-Earths
    • The sun is among the 10% most massive stars in the Milky Way
    – A more comprehensive search would encompass all FGK stars

• HDST allows search of true exo-Earths on a grand scale
  – Thousands of F, G, K stars with sensitivity down to 1 Earth at 1 AU
# Broad Exo-Earth Search with HDST

## Astrometric precision, photon noise (mas)

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| SNR Target | 5.8 | from SIM book, for 1% FAP |
| Slew Time   | 100 sec |


[2] from SIM Book, assuming sun-mass star and earth mass planet:

The semi-amplitude of the angular wobble, \( \alpha \), of a star of a given mass, \( M_\star \), and distance, \( D \), due to a planet of a given mass, \( M_p \), orbiting with a semi-major axis, \( a \), is given by:

\[
\alpha = 3.00 \frac{M_\odot}{M_\star} \frac{M_p}{M_\odot} \frac{a}{(1 \text{ AU})} \frac{1 \text{ pc}}{D} \text{ mas}
\]

The benchmark case is an Earth-mass planet orbiting 1 AU from a solar-mass star located 10 pc away. For such a planet, the astrometric semi-amplitude, \( \alpha \), is 0.3 mas. Using specially chosen and vetted
Also Powerful tool for Targeted Search

- HDST, armed with precision astrometry, can dramatically enhance existing exoplanet science instruments
  - Transits
    - Transits can get size, but not mass
  - Radial Velocity
    - RV searches yield planet masses that are lower limits (M Sin i ambiguity)
  - HDST precision astrometry can
    - characterize exoplanet systems, with masses and orbits (including inclinations) down to the sensitivity needed for Exo-Earths
    - Search all existing G stars within ~ 50 pc (about 2000 G stars) in a little over 1 yr
BACKUP
Example, use 4 dithered undersampled images to generate one nyquist sampled image

Start with asymmetrical PSF

Offset 0 0
Pixelation 1 pix/(lam/D)

Offset 0 0
Pixelation 1 pix/(lam/D)

Offset 0 8
Pixelation 1 pix/(lam/D)

Offset 0 8
Pixelation 1 pix/(lam/D)

Offset 8 0
Pixelation 1 pix/(lam/D)

Offset 8 0
Pixelation 1 pix/(lam/D)

Offset 8 8
Pixelation 1 pix/(lam/D)

Offset 8 8
Pixelation 1 pix/(lam/D)

Offset 8 8
Pixelation 2 pix/(lam/D)

Offset 0 0
Pixelation 2 pix/(lam/D)

Offset 0 0
Pixelation 2 pix/(lam/D)

Offset 0 0
Pixelation 2 pix/(lam/D)
Extreme Focal Plane Calibration (benefits)

- Astrometry at ~ 1~10uas
- **Photometry** at $10^{-4} \sim 10^{-5}$. (can’t do astrometry at 1e-5 pix without photometry)
  - This is *Kepler* level photometry (if photon statistics allow)
- **Correction of asymmetrical PSFs** in images of Galaxies for **Weak Lensing**

- If the spatially varying PSF also changes in time (over weeks/months) that can be tracked as well.
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Focal Plane Array

Ref. 1
Ref. 2
Ref. 3
Ref. 4
Ref. 5
Target

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• A point source diffracts off the focusing optic to produce an airy spot on the detector.

The wavefront from the fiber is near perfect. (geometric point) There is only 1 optic that reimages the fiber to the detector.

That optic is not perfect $\lambda/100$, the airy spot is not exactly the bessel function(squared). But since all the images use the same part of the same focusing optic, all the PSFs are identical.
We noticed ghost images in our setup. At first we looked for stray light reflections (eg window on the detector)

But when we looked at how the ghost images moved relative to the real images, we realized that these ghosts were the result of electrical xtalk between the 4 read amps on the chip.

Our setup at JPL and our colleague’s setup in Grenoble used the same E2V chip but totally different readout electronics. But saw similar ghosts.
• For each pixel’s output, the laser fringes are a near perfect sinusoid in time. The phase of the sinusoid is as good as the RF generator driving the AOM.

• A deviation of a perfect sinewave is a measure of the non-linearity of the detector.

A quick test of photometric nonlinearity is the appearance of a 2\textsuperscript{nd} harmonic. When taking a temporal FFT.

CCDs don’t have (much) persistence. But with fiber illumination we can change the temporal freq without affecting the fringe amplitude. A change in amplitude with freq is a sign of persistence.
Internal Fringes from Laser Illumination

- With CCDs, at long wavelengths, the detectors are semi-transparent.
- A flat field measurement in laser light (especially at long $\lambda$) will exhibit “fringing” that is absent in a white light flat field at the same wavelength.
- The solution is to use a tunable laser, and repeat the detector calibration measurements across enough different wavelengths to average this effect away.

1.2um 7% bw
There are many 100’s of 17 mag stars in the WFIRST FOV and the spatially varying PSF can be measured at every star location.

\[
\text{FFT}(\text{fixed\_image}) = \frac{\text{FFT(image)} \cdot \text{FFT(PSF\_symetric)}}{\text{FFT(PSF\_actual)}}
\]

- PSF asymmetry in images of resolved objects can be removed with conventional fourier deconvolution techniques.
- Sub-pixel calibration becomes more important when the image is undersampled.
Overview of Calibration

• The technology described here is subpixel calibration of detectors, at a level that would enable 10-5 I/D astrometry, but also photometry and correction of field dependent PSF at a similar level.

• Sub-pixel calibration is essential for astrometry accuracy < 10-3 pixels, but it is even more important when the focal plane is NOT Nyquist sampled such as for WFIRST.

• This approach to high accuracy astrometry is different than the approach used where pixelation errors are “averaged” by moving the image across many (1000’s of pixels) The two approaches can be combined (they are not mutually exclusive). In our case we move the images to 10’s of positions instead of 10’s of 1000’s of positions.

• The basic idea is to measure the sub-pixel imperfections of detectors, and develop the math/algorithms to incorporate that calibration data into photometry, astrometry, and shape (psf deconvolution) applications.